

Geometry I, 2006 : Coursework 6

Show your calculations.

1. [2 marks] Determine a Cartesian equation for the plane through the points $(1, -2, 3)$, $(-5, 0, 1)$ and $(2, 1, -1)$.
2. [3 marks] Let $\underline{u}, \underline{v}$ be arbitrary vectors. Simplify the expression $|\underline{u} \times \underline{v}|^2 + (\underline{u} \cdot \underline{v})^2$.
3. [3 marks] Prove that for all vectors $\underline{u}, \underline{v}, \underline{w}$, we have $\underline{u} \times (\underline{v} + \underline{w}) = (\underline{u} \times \underline{v}) + (\underline{u} \times \underline{w})$. (You may assume that for all $\underline{u}, \underline{v}, \underline{w}$, we have $(\underline{u} + \underline{v}) \times \underline{w} = (\underline{u} \times \underline{w}) + (\underline{v} \times \underline{w})$.)
4. (i) [2 marks] Find the volume of the parallelepiped with sides corresponding to $\underline{u}, \underline{v}, \underline{w}$, where $\underline{u} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $\underline{v} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $\underline{w} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$.
(ii) [2 marks] Are $\underline{u}, \underline{v}, \underline{w}$ coplanar? If $\underline{u}, \underline{v}, \underline{w}$ are not coplanar then are they a right-handed triple or a left-handed triple? Justify your answers.
5. [8 marks] Decide whether the following statements are true or false. In each case justify your answer.
 - (i) If $\underline{u}$ and $\underline{v}$ are vectors such that $\underline{u} \times \underline{v} = \underline{0}$, then we must have $\underline{u} = \underline{0}$ or $\underline{v} = \underline{0}$.
 - (ii) If $\underline{u}$ is a vector such that $\underline{u} \times \underline{v} = \underline{0}$ for every vector $\underline{v}$, then we must have $\underline{u} = \underline{0}$.
 - (iii) If $\underline{u} \times \underline{v} = \underline{v} \times \underline{u}$ then $\underline{u} \times \underline{v} = \underline{0}$.
 - (iv) $(\underline{u} \times \underline{v}) \times \underline{w} = \underline{u} \times (\underline{v} \times \underline{w})$ for all vectors $\underline{u}, \underline{v}, \underline{w}$.

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 16th November.

Leonard Soicher, 9 November 2006